THE STAGING OF VENTURE CAPITAL FINANCING: MILESTONE VS. ROUNDS

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Abstract

Venture capital funding is commonly provided to start-up firms on a piecemeal basis over numerous stages. One way in which this can be implemented is through milestone financing, where a venture capitalist commits upfront to providing additional future funding contingent upon the firm meeting certain conditions, or milestones. Alternately, the firm can operate without a firm commitment in place, still reasonably expecting to be able to receive additional rounds of funding after goals are met (round financing).

We identify four dimensions which can affect the optimal contract and choice of financing method: entrepreneurial effort, venture capitalist effort, venture capitalist preference for liquid investments, and heterogeneous expectations about the feasibility of the underlying real technology. The effects of these on the optimal milestone and round financing contracts are examined. Firms that prefer milestone financing to round financing (and conversely) are characterized.

1. Introduction

It is now difficult to envision grooming of technology-based start-up firms without venture capital backing. Venture capital funding has been one of the fastest growing segments of the financial sector. During 2001, U.S. venture capital funds invested 42 billion dollars in early stage investing, compared to only one billion in 1991.[1] From an academic perspective, two features of venture capital are of particular interest compared to other forms of financing. First, venture capital investment is often called smart money, denoting the fact that it plays a dual role. In addition to providing funding, venture capitalists serve their portfolio firms by providing coaching and guidance, as well as networking for strategic alliances and for further funding. Second, unlike investments in quoted companies, there are only a few investors involved in the funding, all of whom are presumed to be sophisticated. Therefore the terms of the funding need not be simple. In fact, they tend to be quite complicated so as to best address the various aspects of each particular case. Even a casual inspection of a typical term sheet reveals a strikingly large number of features, such as convertible and preferred securities, warrants, staged investment with milestones, anti-dilution ratchets, voting arrangements, liquidation preferences, and vesting arrangements.[2]

The current study takes a close look at one phenomenon - the venture capitalist contractual commitment over time to the investment. Two types of financial arrangements are contrasted. The first is milestone financing, in which there is an upfront commitment by the venture capitalist to invest beyond the company's immediate needs. The funding commitment is at a predetermined price and is received by the start-up company once pre-specified technological or operational milestones are met. The second arrangement is round financing, in which there is no

[1] During its peak in the year 2000, U.S. venture capital funds invested 104 billion dollars.

[2] There are clearly other, non-financial, attributes that are unique to investment in start-ups. Most notably, they include the extremely high level of uncertainty (technological, managerial, and untested demand in the

pre-commitment to invest beyond the current time. Therefore, any subsequent investment is priced based on the realization and the status of the start-up company at the time of the subsequent round. We refer to the former arrangement as milestone investment, and the latter as round investment. The paper will characterize the situations where the use of milestones is better, vis-à-vis the circumstances where round investment is superior. In the process we explicitly account for several other key characteristics of investment in start-ups - managerial skill, the effort level of the entrepreneur, and the effort expended by the venture capitalist. Other pervasive features that our model accommodates are differential beliefs about the likelihood success of the start-up firm (the entrepreneur is often more optimistic) and the possibility that the venture capitalist has a preference for liquid investments. As will be elaborated upon later, venture capital funds have a strong incentive to exit their investment sooner rather than later, which we phrase liquidity preference. All these considerations play a role in the relative attractiveness of milestone financing compared to round financing. For instance, whether the entrepreneurial effort becomes more or less important when technology succeeds can make a pivotal difference to the optimal financial arrangement.

In a comprehensive study, Kaplan and Strömberg (2003) document the features of venture capital contracts. They analyze the terms sheets of over 200 rounds of venture backed investments and link the statistics to agency problems. Sahlman (1990) and Lerner (1995) provide evidence on the dual role of venture capitalists and their involvement in monitoring and governance. Subsequently, Hellman and Puri (2000, 2002) statistically confirm that the in-kind services of venture capitalists are of economic significance, through a reduction in time to bring a product to market and by professionalizing the start-up company. Also on the empirical side, Gompers (1995) provides detailed statistics of venture capital investments and explores factors that influence the amount invested in a round and the duration between rounds.

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market), practically no history to analyze, early dependency on the entrepreneur, and lack of tangible assets. These factors intensify the need to use complicated financial contracts.

From an analytical perspective, Repullo and Suarez (1998) and Schmidt (1999) model the advisory role of the venture capitalists within a double-sided moral hazard framework which gives rise to features in convertible securities used in venture capital financing. Cornelli and Yosha (2003) show that the use of convertible securities mitigates the incentive of entrepreneurs to engage in window dressing practices.[3] The rationale for the advisory role of venture capitalist is analyzed in Casamatta (2003). Under moral hazard, if the entrepreneurial effort is more efficient (less costly) than that of the consultant, the latter is not hired unless invested financially in the project, in the spirit of venture capital involvement. Chemmanur and Chen (2001) study an opposite paradigm whereby pure financing (angel investing) is contrasted with active involvement of venture capital investment. The desired form of financing is characterized based on factors such as scarcity of venture capital funding. Finally, Cossin, Leleux and Saliasi (2002) use a real options paradigm to analyze the pricing impact of covenants such as priority in liquidation, staging, and convertibility. However, their approach takes as given the cash flow of the firm and so does not endogenize the security design choice.[4]

In comparing milestone financing with round financing, we identify four primary differences, giving rise to a relative advantage for each form of financing under certain conditions. First, because each round is contracted separately, round financing implies that the price of corporate claims sold at each financing stage should be set at a competitive level (conditional on common knowledge at that point in time). In contrast, milestone financing contracts multiple stages of financing simultaneously, giving the two contracting parties the ability to adjust the relative prices, and therefore the magnitudes, of the parties' claims on the firm across different outcomes. This may be advantageous in generating better incentives for the contracting parties to undertake costly effort to increase the value of the firm. In particular, if

[3] Other analytical models that rationalize the use of convertible preferred stocks over a mix of debt and equity are Bascha (2001) and Houben (2002).

either the entrepreneur or venture capitalist can undertake private effort to increase corporate cash flows, it is well known that the principal-agent problem cannot result in first-best effort being undertaken (except in the extreme case where the party owns the rights to all marginal increases in corporate cash flows). Thus, in the context of entrepreneurial and/or venture capitalist effort, milestone financing can offer a valuable flexibility unavailable with round financing.

Second, the level of upfront commitment differs between milestone financing and round financing. At the first stage, round financing achieves a financing commitment for that stage alone. Milestone financing, in contrast, raises capital for the first stage, with the promise of additional funds in the future (conditional on achievement). Since the financier accepts a higher level of financing commitment under milestone financing, she should receive a commensurately larger claim on corporate cash flows, while the entrepreneur has a smaller claim on corporate cash flows. Again, in the context of effort that can be undertaken by the contracting parties, milestone and round financing result in different incentives for those parties to undertake effort.

Third, we consider the effects of heterogeneity of expectations about the likelihood of success of the real technology underlying the start-up firm. Of all possible real projects that could be undertaken, those expected to be most profitable should be taken on. Thus, an entrepreneur undertaking a start-up will naturally be optimistic about his probability of success, perhaps even more so than their financiers. This gives rise to differential beliefs about the likelihood of various outcomes. Indeed, one of the objectives achieved through venture capital term sheets is to accommodate more optimistic entrepreneurs by committing to grant them more shares if promises are fulfilled and milestones are met (Kaplan and Strömberg 2003). Naturally, an optimal contract will tend to tilt toward giving contingent claims over possible states to the party who places the highest probability on that state.

[4] Hsu (2002) also uses contingent claims analysis to study the consequences of staging. The premise of his model, however, is that the entrepreneur does not seek to maximize expected value but seeks instead to maximize the probability of getting funded in the next financing round.

Fourth, we consider the effect of liquidity. Because venture capitalists raise their funds through limited partnerships which have a finite time horizon, they strongly prefer an exit from each investment before the end of the fund life. Venture capitalists also report performance in terms of IRR and so they prefer to return money to investors earlier than later. States which result in a possible sale or public offering (or other liquifying event) of the firm will be of particular interest for the venture capitalist. This has an effect similar to heterogeneity of beliefs: an optimal contract will tend to tilt toward giving the venture capitalist contingent claims in states leading to a liquidity event for the firm. As mentioned previously, milestone financing allows additional flexibility in fashioning a contract. In the context of belief heterogeneity or venture capitalist liquidity preference, milestone financing allows tilting of the contract to respond to the above preferences of the contracting parties.

These effects may interact with one another. For example, the flexibility in milestone financing may be valuable because of incentive effects, and also because of differential preferences across states by the contracting parties. The induced tilt in the optimal ownership of contingent claims generated by belief heterogeneity runs counter to that generated by venture capitalist liquidity preference. Furthermore, their combined effect may either reinforce or counter the induced tilt generated by incentive effects.

We compare the results of milestone and round financing in various scenarios. For example, when the only role of the entrepreneur is skill (rather than effort), the need for contract flexibility dominates, and we show milestone financing is preferred to round financing. With homogenous expectations and no liquidity preference, flexibility is unimportant if the venture capitalist effort is relatively inconsequential and technological success leaves unchanged the relative importance of entrepreneurial effort. Therefore, incentivizing the entrepreneur dominates the contractual relationship and round financing is preferred to milestone financing. Marginalizing the role of effort by both parties (so that only managerial skill is important), either

heterogeneous beliefs or venture capitalist liquidity preference makes milestone financing preferred over round financing.

The basic model is introduced in Section 2. Milestone financing, with entrepreneurial and venture capitalist efforts, and with homogeneity of beliefs and no liquidity effects, is discussed in Section 3. The analogous discussion for round financing is in Section 4. Section 5 studies in isolation the effect of venture capitalist effort, then of entrepreneurial effort. Heterogeneous beliefs and liquidity are discussed in Section 6, and Section 7 concludes.

2. The model

The real technology of the firm has two possible outcomes, "failure" and "success", determining two possible states, indexed by $i \in \{1, 2\}$ respectively. Both states may result in positive cash flow for the firm; technological "failure" of state 1 denotes lower expected cash flow relative to technological "success" of state 2.

The outcome of the technology is determined and observed after the firm spends I cash at time 0. If the technology is successful, the firm spends an additional J cash at time 1 to achieve the higher expected cash flow associated with success. The firm generates a single cash flow at time 2, whose expectation depends partially upon the outcome of the technology. The first I dollars of corporate spending can be interpreted as initial R&D or product development, while success early in the firm life requires additional corporate spending J to achieve growth or to take the project to the next level.

After the initial I has been financed and spent by the firm, the outcome of the technology is publicly revealed. The entrepreneur then has the opportunity to expend personally costly effort to increase expected corporate cash flow. At time 1, conditional on the technology being successful, the firm raises and spends an additional J.[5] The venture capitalist then observes the

[5] Technological success thus allows the firm to be taken "to the next level," investing additional funds to receive higher future revenue. For example, success could be interpreted as successful product

managerial skill level. If it is low, the venture capitalist has the ability to expend personally costly effort to increase managerial skill. In practice, this can take many forms, such as coaching and mentoring management to achieve a better professionalism, stepping in to replace management, conducting an external search for a new management, helping management "learn the ropes" if they lack the necessary skills applicable to a start-up firm or the appropriate industry, providing guidance through operations on the board of directors, and networking in the financial community in preparation for future syndication (e.g., Lerner 1995 and Hellmann and Puri 2002). All of these require commitments of time, of which the venture capitalist has a limited amount available. At this time, the venture capitalist also learns how costly increasing managerial skill would be. After the venture capitalist expends her desired effort, the firm generates a cash flow, which is then distributed to the entrepreneur and venture capitalist as per earlier agreements.

The magnitude of the expected cash flow of the firm depends upon the resolution of the real technology, entrepreneurial effort, and managerial skill. The cash flow of the firm, generated at time 2, is either unity or zero. The probability of achieving a cash flow of unity is $(aV_i + bU_i)$, where i indexes the technology state, $a \geq 0$ is the entrepreneurial effort, $b \in \{0, 1\}$ is the managerial skill level, while V_i and U_i are state-dependent sensitivities of entrepreneurial effort and managerial skill, with $V_2 \geq V_1 \geq 0$ and $U_2 \geq U_1 \geq 0$.

Under milestone financing, the entrepreneur and venture capitalist agree to a contract at time 0 wherein the venture capitalist immediately supplies I cash to the firm, receiving the right to a claim (fractional share) $f_1 \in [0, 1]$ of the cash flow of the firm. It is simultaneously agreed that if the milestone is met (the technology is a success, reaching state 2), the venture capitalist will supply an additional J cash to the firm at time 1, increasing her share of the firm to $f_2 \in [f_1,$

development, which naturally leads to spending additional funds in marketing a product. Alternatively, success could be developing a successful technology or process, which naturally leads to spending additional funds to develop commercial applications of the technology. Of course, "success" should imply that the incremental expected cash flow to the firm in state 2 should be greater than the incremental investment J.

1]. Thus, if technology is a failure (the firm is in state 1), the venture capitalist receives a share f_1, while the entrepreneur receives the residual share $(1 - f_1)$. If technology is a success (the firm is in state 2), the venture capitalist supplies an additional J cash and receives share f_2 while the entrepreneur receives share $(1 - f_2)$.[6]

Alternatively, under round financing, the entrepreneur and venture capitalist agree to a contract at time 0 (the first round) wherein the venture capitalist immediately supplies I cash to the firm, receiving the right to a share $F_1 \in [0, 1]$ of the cash flow of the firm. If the technology subsequently is a success, necessitating additional J in financing at time 1, the entrepreneur and venture capitalist enter the second financing round. In that financing round, the venture capitalist will agree to supply an additional J cash to the firm; in return, the venture capitalist's stake in the firm is raised to $F_2 \in [F_1, 1]$ determined by negotiations at that time.

The difference between milestone and round financing is the following. Under milestone financing, the firm gets a commitment and terms at time 0 for all needed financing: I at time 0, and J at time 1 (conditional on technological success). Under round financing, the firm gets a commitment and terms at time 0 only for immediate financing needs: I at time 0. If technological success occurs, the firm negotiates terms at time 1 for an additional J in financing.[7]

After the first stage of financing I is invested, and subsequent resolution and revelation of technological uncertainty occurs, the entrepreneur has the opportunity to expend effort a_i (chosen effort typically depends upon the state). The personal cost to the entrepreneur of expending effort a_i is $A(a_i)^2/2$, where $A > 0$. Note that entrepreneurial effort affects the probabilities of the possible cash flow outcomes. Thus, an outsider cannot perfectly infer entrepreneurial effort expended. In

[6] Since only the realizations of the cash flow and the technology are observable, these are the most general possible contracts with non-negative payouts.

[7] We distinguish between stages of financing and rounds of financing as follows. Stages of financing refer to the points in time when additional cash is received by the firm. Rounds of financing refer to stages with accompanying negotiation of financing terms. Thus, when the real technology is successful, both milestone and round financing will have two stages in our model, while only round financing will have two rounds.

this spirit, it is assumed that entrepreneurial effort is observable [by the venture capitalist] but not contractible.[8]

After entrepreneurial effort is expended, and (if needed) any subsequent raising and spending of capital J in the second stage of financing, the venture capitalist observes managerial skill b. Managerial skill is binomial, equaling 0 with probability π, and 1 with probability $(1 - \pi)$, independent of the state. Managerial skill and entrepreneurial effort are different (and independent) variables in the model. Entrepreneurial effort reflects the willingness to work hard, possibly long hours, especially early in the life of the firm, to establish the business. Managerial skill (the manager and entrepreneur might not be the same person) reflects the ability of the manager to solve problems faced by the business organization as it grows. As the primary role of the manager is to enable an organization to function smoothly, the managerial skill variable may also measure the intrinsic structure of the corporate organization with respect to its ability to operate and overcome business difficulties.

If the managerial skill is 0, the venture capitalist can expend effort, at a cost to herself of c, to increase the skill to 1. Because managerial shortcomings may occur over many dimensions, and may range from being relatively painless to solve through presenting virtually intractable problems, c is a random variable whose value is observed by the venture capitalist at the same time she observes managerial skill. It is assumed that c is distributed uniformly over [0, K], with $K > U_2$, and that c is independent of b and the technology state.

The following is the timeline for events. At time 0, a financing contract is specified. This may be a milestone contract over both stages, or a round contract for the first stage of financing. Either way, the firm raises (and spends) I cash. Uncertainty about the technology state i is fully resolved and revealed. The entrepreneur chooses a level of effort a_i (observable to the venture capitalist but not contractible). At time 1, if the technology is a success, the firm raises (and

[8] In particular, this precludes a third type of contract, similar to milestone, wherein the share price in the second stage is explicitly conditioned on entrepreneurial effort.

spends) J cash at the second financing stage. With milestone financing, these funds are raised under the already agreed terms. With round financing, the terms are agreed at this time. Next, managerial skill and the cost of improving managerial skill are observed by the venture capitalist. The venture capitalist chooses whether to undertake effort at cost c to affect management skill b. At time 2, the firm generates a cash flow (of either unity or zero), which is distributed between entrepreneur and venture capitalist based on the earlier contracting.

At time 0, both entrepreneur and venture capitalist assign probabilities p_1 and p_2 to the two states (technological failure and success, respectively).[9] All parties are risk-neutral. The venture capitalist acts competitively in pricing contracts. The discount rate is zero.

3. Milestone financing

Determining the optimal milestone financing contract requires backward induction. First venture capitalist effort, then entrepreneurial effort, must be determined. Effort levels will depend on the particular milestone financing contract characterized by the shares (f_1, f_2).

In determining whether to undertake effort to improve managerial skill, the venture capitalist compares her claim of corporate cash flow with or without effort against her personal cost of effort. Since her expected claim on corporate cash flow in state i is f_i (a_i V_i + b U_i), she undertakes effort when b = 0 and $c < f_i U_i$.

At time 1, before the venture capitalist has learned the manager's skill and her own cost of effort, the expected level of managerial skill (post-venture capitalist effort) in state i is given by

$$b_i = (1 - \pi) + \pi \cdot \text{Prob}(c < f_i U_i) = 1 - \pi + \pi f_i U_i / K. \qquad (1)$$

[9] This will be relaxed in Section 6, where the effects of heterogeneous beliefs about technological success are considered.

This is state-dependent because venture capitalist effort is state-dependent. The corresponding expected cost of effort to the venture capitalist in state i is

$$c_i = \pi \cdot \text{Prob}(c < f_i U_i) \cdot E(c \mid c < f_i U_i) = \pi f_i^2 U_i^2 / 2K.$$

Clearly, the likelihood of expending effort and the expected cost of effort in state i is increasing in the venture capitalist's share f_i of the firm.

The model allows for the possibility of an outcome known as the **living dead**, which describes a firm expecting to generate cash flows, and thus worthwhile to keep operating, yet not worthwhile for the venture capitalist to expend effort to improve. The probability of a firm becoming living dead, conditional on technology state i, is given by $1 - b_i$. Since $f_2 \geq f_1$ and $U_2 \geq U_1$, equation (1) implies that $(1 - b_1) \geq (1 - b_2)$, so under milestone financing, the firm is less likely to become living dead if the technology is successful.

In determining his optimal level of effort, the entrepreneur maximizes his share of expected corporate cash flow less his personal cost of effort.

$$\text{Max} \quad (1 - f_i)(a_i V_i + b_i U_i) - A a_i^2 / 2.$$

$$a_i \geq 0$$

Optimal entrepreneurial effort is $a_i = (1 - f_i)V_i / A$ in state i, increasing in his firm share $(1 - f_i)$.

A feasible milestone contract must offer the venture capitalist a claim on the firm large enough to reimburse her for providing funds (while recognizing her cost of undertaking effort). The milestone contract is a commitment for the venture capitalist to provide I at the first stage, and J at the second stage (conditional on technological success), for an expected $I + p_2 J$ in total capital provided. Feasible contracting requires

$$I + p_2 J = \sum_i p_i [f_i(a_i V_i + b_i U_i) - c_i],$$

$$= \langle f_i(1 - f_i)V_i^2\rangle/A + (1 - \pi)\langle f_i U_i\rangle + (\pi/2K)\langle f_i^2 U_i^2\rangle, \quad (2)$$

after substituting for a_i, b_i, and c_i. The triangular brackets indicate an expectation, with weights p_i, taken over the two states.[10]

The optimal contract maximizes the value of the entrepreneur's residual claim at time 0, less the cost of entrepreneurial effort,

$$\Phi_M = \sum_i p_i [(1 - f_i)(a_i V_i + b_i U_i) - A a_i^2/2],$$

$$= \langle (1 - f_i)^2 V_i^2\rangle/2A + (1 - \pi)\langle (1 - f_i)U_i\rangle + (\pi/K)\langle f_i(1 - f_i)U_i^2\rangle,$$

after substituting for a_i and b_i. Substituting from the venture capitalist feasibility constraint (2), the optimal milestone contract solves

$$\text{Max} \quad \Phi_M = \langle (1 - f_i^2)V_i^2\rangle/2A + (1 - \pi)\langle U_i\rangle + (\pi/2K)\langle [1 - (1 - f_i)^2]U_i^2\rangle - (I + p_2 J), \quad (3)$$

$$0 \le f_1 \le f_2 \le 1$$

$$\text{subject to:} \quad I + p_2 J = \langle f_i(1 - f_i)V_i^2\rangle/A + (1 - \pi)\langle f_i U_i\rangle + (\pi/2K)\langle f_i^2 U_i^2\rangle.$$

Solution details are given in the Appendix.

4. Round financing

Determining the optimal round financing contract proceeds similarly to the case of milestone financing. At time 1, (after the second financing round, if the technology state 2 occurs), the venture capitalist owns a share F_1 or F_2 of the firm, depending upon the technology state.

The venture capitalist undertakes effort in state i when managerial skill is b = 0 and when cost $c < F_i U_i$. Before the venture capitalist knows the managerial skill and her cost of effort, the

expected managerial skill level b_i (post-venture capitalist effort) and the expected cost of venture capitalist effort c_i in state i are

$$b_i = (1 - \pi) + \pi \cdot \text{Prob}(c < F_i U_i) = 1 - \pi + \pi F_i U_i / K, \quad (4)$$

$$c_i = \pi \cdot \text{Prob}(c < F_i U_i) \cdot E(c \mid c < F_i U_i) = \pi F_i^2 U_i^2 / 2K.$$

Since $F_2 \geq F_1$ and $U_2 \geq U_1$, equation (4) implies that $(1 - b_1) \geq (1 - b_2)$, so under round financing, the firm is less likely to become living dead if the real technology is successful. We summarize this and our earlier result on the living dead in the following.

Proposition 0. *Under either milestone financing or round financing, the firm is less likely to become living dead if the real technology is successful.*

The share of the firm received by the venture capitalist in the second financing round is priced to reflect both additional financing contributed and additional effort the venture capitalist will undertake. (Her expected effort increases because she now has a larger stake in the firm and thus recaptures more of her effort through her claim on corporate cash flow.) If the venture capitalist did not invest in the second round, a third party would need to provide capital J. Assuming the outside party's claim dilutes both the entrepreneur's and venture capitalist's claims proportionately, competitive pricing implies

$$F_2(a_2 V_2 + b_2 U_2) - c_2 - J = F_{2D}(a_2 V_2 + b_{2D} U_2) - c_{2D},$$

where F_{2D} would be the diluted venture capitalist claim, $b_{2D} = 1 - \pi + \pi F_{2D} U_2 / K$ the expected managerial skill level (post-venture capitalist effort), and $c_{2D} = \pi F_{2D}^2 U_2^2 / 2K$ the expected cost of

[10] We assume that the claim the venture capitalist receives exactly compensates her for providing funds and

venture capitalist effort resulting if a third party invested in the second round in technology state 2. Denote that third party claim by G, so $F_{2D} = (1 - G)F_1$, and the third party breakeven constraint is $G(a_2 V_2 + b_{2D} U_2) = J$. The competitive pricing constraint then implies that, when investing, the venture capitalist's share F_2 is determined by

$$[a_2 V_2 + (1 - \pi)U_2]F_2 + (\pi U_2^2/2K)F_2^2 = J + [a_2 V_2 + (1 - \pi)U_2]F_{2D} + (\pi U_2^2/2K)F_{2D}^2, \qquad (5)$$

where F_{2D} satisfies $F_{2D} = F_1 - F_1 J/[a_2 V_2 + (1 - \pi)U_2 + \pi F_{2D} U_2^2/K]$.

Entrepreneurial effort in state 1 is determined by

$$\text{Max} \quad (1 - F_1)(a_1 V_1 + b_1 U_1) - A a_1^2/2 ,$$
$$a_1 \geq 0$$

which implies optimal effort $a_1 = (1 - F_1)V_1 /A$. Determining optimal entrepreneurial effort in state 2 is more complicated. In state 2, the entrepreneur realizes that his effort impacts firm value, and therefore affects the second round share price, and therefore F_2, as well as future venture capitalist effort. Entrepreneurial effort in state 2 is determined by

$$\text{Max} \quad (1 - F_2)(a_2 V_2 + b_2 U_2) - A a_2^2/2 \qquad (6)$$
$$a_2 \geq 0$$
subject to $F_2(a_2)$ determined by (5).

Details of solving (6) for a_2 are in the Appendix. In addition to (5), feasible contracting for the venture capitalist requires

$$I + p_2 J = \sum_i p_i [F_i(a_i V_i + b_i U_i) - c_i]$$

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expected effort.

$$= \langle F_i a_i V_i \rangle + (1 - \pi)\langle F_i U_i \rangle + (\pi/2K)\langle F_i^2 U_i^2 \rangle, \quad (7)$$

The residual value to the entrepreneur at time 0 can be written as

$$\begin{aligned}
\Phi_R &= \sum_i p_i [(1 - F_i)(a_i V_i + b_i U_i) - A a_i^2/2] \\
&= \sum_i p_i [(1 - F_i)(a_i V_i + b_i U_i) - A a_i^2/2] + \sum_i p_i [F_i(a_i V_i + b_i U_i) - c_i] - (I + p_2 J) \\
&= \langle a_i V_i \rangle + (1 - \pi)\langle U_i \rangle + (\pi/2K) \langle [1 - (1 - F_i)^2] U_i^2 \rangle - (A/2)\langle a_i^2 \rangle - (I + p_2 J).
\end{aligned}$$

Thus, the round financing contract is specified by

$$\Phi_R = \langle a_i V_i \rangle + (1 - \pi)\langle U_i \rangle + (\pi/2K) \langle [1 - (1 - F_i)^2] U_i^2 \rangle - (A/2)\langle a_i^2 \rangle - (I + p_2 J), \quad (8)$$

subject to: $I + p_2 J = \langle F_i a_i V_i \rangle + (1 - \pi)\langle F_i U_i \rangle + (\pi/2K)\langle F_i^2 U_i^2 \rangle$,

equation (5),

and the first-order condition from (6). This determines the round financing contract.

5. Separating the effects of venture capitalist and entrepreneur effort

Much insight can be gained into the desirability of the financing alternatives of Sections 3 and 4 by considering separately the effects resulting from the abilities of the venture capitalist and the entrepreneur to undertake effort to improve the firm. The first subsection considers the case where the role of the entrepreneur is limited to contributing skill, so his effort level is not important. Venture capitalist effort is still assumed to play a role in the model. In the subsequent subsection we examine the case where the entrepreneur contributes both skill and effort, but

assume that the venture capitalist effort is not important, i.e., her contribution is limited to the infusion of capital.

The intuition for our results is as follows. The advantage associated with milestone financing is contract flexibility.[11] With round financing, funds are raised at each stage based upon the fair value of the firm at that point. However, with milestone financing, a contract can be written assigning claims to each party over multiple possible future outcomes (technological success or failure). These claims need not be designed so that they are *ex post* priced fairly in each possible outcome; they need only be designed so that they are *ex ante* priced fairly, before knowing the outcome. This gives additional flexibility to milestone contracting relative to round financing. Thus, a milestone contract could be written to give one party a relatively disproportionate claim in one state, as long as the other party is appropriately compensated by adjusting their claim in the other state.

The advantage associated with round financing is increased entrepreneurial incentive. Because round financing has a lower upfront commitment than milestone financing, consisting only of a commitment to finance the current stage, the venture capitalist receives lower compensation upfront. Therefore, the entrepreneur captures more of any increase in the firm value between the first and second financing stages, resulting in an increased incentive to expend personally costly effort.

For a particular firm, the preferred method of contracting, whether milestone financing or round financing, is determined by whether the advantage associated with milestone financing (flexibility) outweighs the advantage associated with round financing (increased entrepreneurial incentive). With homogeneous beliefs about the probability of technological success and no venture capitalist liquidity preference, the greater advantage partially depends upon the relative importance of entrepreneurial effort and venture capitalist effort, as examined in Subsections 5A

[11] Somewhat paradoxically, because milestone financing involves earlier commitment, it offers more flexibility than round financing.

and 5B. Additional effects resulting from heterogeneous beliefs and/or venture capitalist liquidity preference are explored in Section 6.

5A. Managerial skill and venture capitalist effort only

This subsection considers the case where there is only managerial skill and venture capitalist effort, while the level of entrepreneurial effort is not materially important. The mathematics are considerably simplified. The simplification can be seen by letting the entrepreneurial effort cost parameter A increase unboundedly in the derivations of Sections 3 and 4. As before, optimal venture capitalist effort implies $b_i = 1 - \pi + \pi f_i U_i/K$, and $c_i = \pi f_i^2 U_i^2/2K$ under milestone financing, while $b_i = 1 - \pi + \pi F_i U_i/K$, and $c_i = \pi F_i^2 U_i^2/2K$ under round financing. Entrepreneurial effort becomes $a_i = 0$ under either milestone or round financing.

Under milestone financing, the entrepreneur's residual value optimization (3) simplifies to

$$\text{Max} \quad \Phi_M = (1 - \pi)\langle U_i\rangle + (\pi/2K)\langle[1 - (1 - f_i)^2]U_i^2\rangle - (I + p_2 J), \tag{9}$$

$$0 \le f_1 \le f_2 \le 1$$

$$\text{subject to:} \quad I + p_2 J = (1 - \pi)\langle f_i U_i\rangle + (\pi/2K)\langle f_i^2 U_i^2\rangle.$$

Under round financing, the entrepreneur's residual value (8) simplifies to

$$\Phi_R = (1 - \pi)\langle U_i\rangle + (\pi/2K)\langle[1 - (1 - F_i)^2]U_i^2\rangle - (I + p_2 J), \tag{10}$$

$$\text{subject to:} \quad I + p_2 J = (1 - \pi)\langle F_i U_i\rangle + (\pi/2K)\langle F_i^2 U_i^2\rangle,$$

$$J = (F_2 - F_{2D})(1 - \pi)U_2 + (F_2^2 - F_{2D}^2)\,\pi U_2^2/2K,$$

where F_{2D} satisfies the quadratic $F_{2D} = F_1 - F_1 J/[(1 - \pi)U_2 + \pi F_{2D} U_2^2/K]$. Comparison shows that (9) and (10) are identical (with a transparent renaming of share variables), except that round financing imposes an additional constraint. In this case, milestone financing offers additional

flexibility in the choice of the division of the firm across the two states, with no downside. Therefore, milestone financing dominates round financing in the absence of entrepreneurial effort.

Proposition 1. *When there is only managerial skill and venture capitalist effort possible, milestone financing is preferred to round financing.*

5B. Managerial skill and entrepreneurial effort only

This subsection derives the special case where venture capitalist effort is not possible, so there is only managerial skill and entrepreneurial effort. As in the previous subsection, the mathematics are considerably simplified. (This is especially true under round financing because, although entrepreneurial effort affects second round dilution, it no longer affects venture capitalist effort.) The simplification can be seen by letting K increase unboundedly in the derivations of Sections 3 and 4. Since there is no venture capitalist effort, $b_i = 1 - \pi$ and $c_i = 0$ for both states, under either financing choice.

Under milestone financing, the entrepreneur's residual value optimization (3) simplifies to

$$\text{Max} \quad \Phi_M = \langle(1 - f_i^2)V_i^2\rangle/2A + (1 - \pi)\langle U_i\rangle - (I + p_2 J), \qquad (11)$$
$$0 \le f_1 \le f_2 \le 1$$
$$\text{subject to:} \quad I + p_2 J = \langle f_i(1 - f_i)V_i^2\rangle/A + (1 - \pi)\langle f_i U_i\rangle.$$

Under round financing, entrepreneurial effort equals $a_1 = (1 - F_1)V_1/A$ in state 1, and substituting from (5), maximizes

$$\text{Max} \quad (1 - F_2)[a_2 V_2 + (1 - \pi)U_2] - Aa_2^2/2 = (1 - F_1)[a_2 V_2 + (1 - \pi)U_2 - J] - Aa_2^2/2,$$
$$a_2 \ge 0$$

satisfied by $a_2 = (1 - F_1)V_2/A$, in state 2. Entrepreneurial effort is determined by the entrepreneur's share of the firm before any dilution. Since entrepreneurial effort affects firm value in the second financing round, the entrepreneur fully recaptures any share dilution in second round pricing. The entrepreneur's residual value simplifies from (8) to

$$\Phi_R = (1 - F_1^2)\langle V_i^2\rangle/2A + (1 - \pi)\langle U_i\rangle - (I + p_2J), \tag{12}$$
$$\text{subject to:} \quad I + p_2J = (1 - F_1)\langle F_iV_i^2/A\rangle + (1 - \pi)\langle F_iU_i\rangle,$$
$$(1 - F_1)J = (F_2 - F_1)[(1 - F_1)V_2^2/A + (1 - \pi)U_2].$$

We show in the Appendix that the values of F_1 and Φ_R in (12) are the same (although F_2 is not) as the values F_1 and Φ_R in the related (13):

$$\Phi_R = \langle(1 - F_i^2)V_i^2\rangle/2A + (1 - \pi)\langle U_i\rangle - (I + p_2J), \tag{13}$$
$$\text{subject to:} \quad I + p_2F_1J = \langle F_i(1 - F_i)V_i^2\rangle/A + (1 - \pi)\langle F_iU_i\rangle,$$
$$F_1 = F_2.$$

In comparing (11) and (13), they are identical (with a renaming of share variables) except for two effects. First, milestone financing offers additional flexibility in that (13) contains an additional constraint. This is the advantage associated with milestone financing. Second, round financing requires less of an upfront commitment: while the venture capitalist commits to an expected $(I + p_2J)$ across two stages under milestone financing, the corresponding value in (12) is only $(I + p_2F_1J)$. Thus, under round financing, a smaller share can be promised to the venture capitalist early on in the project life, leaving a larger share for the entrepreneur. Since the early project life is the critical time for entrepreneurial effort, a larger share of the firm owned by the entrepreneur at that time helps moderate the principal-agent problem and achieve closer to the first-best effort. This is the advantage associated with round financing.

To get a feel for when one of these effects dominates the other, consider various sets of parameter values. First, consider the extreme case of $p_2J = 0$. Then the constraint of (11) and the first constraint of (13) are identical. Under both financing methods, the venture capitalist commits only to financing I at time 0. Milestone financing still offers more flexibility in contract choices, so milestone financing is preferred to round financing here.

Next, suppose $p_2J > 0$. Optimization (11) can be solved using Lagrange multipliers. It is straightforward to show that if $U_2/U_1 > V_2^2/V_1^2$, then the optimal milestone contract satisfies

$$f_2/f_1 = [1 + (1 - \pi)AU_2/V_2^2]/[1 + (1 - \pi)AU_1/V_1^2] > 1.$$

For $U_2/U_1 \leq V_2^2/V_1^2$, the optimal milestone contract has $f_1 = f_2$, and (11) simplifies to

$$\Phi_M = (1 - f_1^2)\langle V_i^2\rangle/2A + (1 - \pi)\langle U_i\rangle - (I + p_2J), \qquad (14)$$

$$\text{subject to:} \quad I + p_2J = f_1(1 - f_1)\langle V_i^2\rangle/A + (1 - \pi)f_1\langle U_i\rangle,$$

$$f_1 = f_2.$$

In comparing (13) and (14), both are effectively one-dimensional problems (the first constraints determine F_1 and f_1, respectively). The only difference is that the left-hand sides of the first constraint differ. This implies that $F_1 < f_1$, so round financing dominates.[12] Because the values of f_1 and f_2 are identical, the flexibility offered by milestone financing allowing quite different values of f_1 and f_2 is not valuable here, so the round financing advantage dominates, and round financing is preferred to milestone financing.[13]

To interpret the comparison between U_2/U_1 and V_2^2/V_1^2, note that U_2/U_1 measures the increase in the sensitivity of corporate cash flow to managerial skill, due to technological success.

[12] The right-hand sides of the constraint are monotonically increasing in f_1 (or F_1) in the relevant area; the value p_2F_1J is therefore achieved with an F_1 lower than the f_1 that achieves the value p_2J.

[13] Implementing the optimal milestone contract here involves the venture capitalist increasing her position by a small amount at a very high price (to approximate $f_1 = f_2$). However, the optimal milestone contract is inferior to round financing.

Thus, U_2/U_1 will be relatively large when technological success increases the importance of managerial skill in determining firm value. Similarly, V_2^2/V_1^2 measures the increase in the sensitivity of corporate cash flow to entrepreneurial effort, due to technological success. Thus, V_2^2/V_1^2 will be relatively large when technological success increases the importance of entrepreneurial effort in determining firm value. We can then interpret $U_2/U_1 > V_2^2/V_1^2$ as saying that technological success increases the importance of managerial skill, relative to entrepreneurial effort. Similarly, $U_2/U_1 \leq V_2^2/V_1^2$ says that technological success increases or leaves unchanged the importance of entrepreneurial effort relative to managerial skill. We can now summarize the results of this subsection.

Proposition 2. *Suppose only managerial skill and entrepreneurial effort are possible.*

- *When technological success increases or leaves unchanged the relative importance of entrepreneurial effort, round financing is preferred to milestone financing.*
- *When technological success decreases the relative importance of entrepreneurial effort, then for sufficiently small probability of technological success or for sufficiently small second stage financing needs, milestone financing is preferred to round financing.*

6. Heterogeneous beliefs and liquidity preference

This section considers the effects resulting from the entrepreneur and venture capitalist placing different values on claims arising from the two possible states. There are two ways in which we allow this to occur. The entrepreneur and venture capitalist may have different expectations (at time 0) of the probability for technological success. One might naturally expect that a venture capitalist willing to enter into a partnership with the entrepreneur to be optimistic about the probability of success, relative to other potential venture capitalists who elected not to become involved with the project. However, the entrepreneur is likely to be more optimistic still. Let p_2 be the probability the entrepreneur assigns to technological success, and q_2 be the probability the

venture capitalist assigns to technological success, with $p_2 \geq q_2$. Thus, at time 0, the entrepreneur and venture capitalist may be willing to pay different amounts for a claim of one dollar contingent on state i occurring.

Another possibility is recognizing the venture capitalist's preference for liquidity. As previously noted, venture capitalists raise funds through limited partnerships with a finite life. This generates a strong preference for early exit from each investment, before the end of the fund life. Thus, the venture capitalist may be hesitant to commit funds to a long-lived enterprise.[14] However, if the corporate technology is successful, it is more likely that the firm may become more liquid (for example, the firm is more likely to be able move to an initial public offering, or the firm is more likely to be taken over by another company for cash or publicly traded equity). We model this by having the venture capitalist value one dollar of [illiquid] assets of the firm in state 1 at value $L \leq 1$, while fully valuing one dollar of [liquid] assets in state 2.

To some extent, the effects of heterogeneous beliefs and liquidity preference will be offsetting. Consider a pair of contingent claims, each paying off one dollar in state i. Relative to the entrepreneur's valuation, the liquidity effect is for the venture capitalist to prefer claims in state 2 (valued for being more liquid), while the heterogeneity effect is for the venture capitalist to prefer claims in state 1 (the entrepreneur is even more optimistic than the venture capitalist about the likelihood of technological success).

To clarify, suppose there is neither the opportunity for entrepreneurial effort nor venture capitalist effort, but there is heterogeneous belief and a venture capitalist liquidity preference. Then $a_i = 0$, $b_i = 1 - \pi$, and $c_i = 0$.

Under milestone financing, the venture capitalist feasibility constraint is

$$I + q_2 J = L q_1 f_1 (1 - \pi) U_1 + q_2 f_2 (1 - \pi) U_2,$$

[14] Thus, liquidity preference is generated by the venture capitalist having a shorter investment horizon than the entrepreneur.

and the entrepreneur's residual value optimization is

$$\text{Max } \Phi_M = \sum_i p_i(1 - f_i)(1 - \pi)U_i = (1 - \pi)\langle U_i\rangle + Z(1 - \pi)f_1U_1 - (p_2/q_2)(I + q_2J), \quad (15)$$
$$0 \le f_1 \le f_2 \le 1$$
$$\text{subject to:} \quad I + q_2J = Lq_1f_1(1 - \pi)U_1 + q_2f_2(1 - \pi)U_2,$$

where $Z = [Lp_2(1 - q_2) - q_2(1 - p_2)]/q_2$ captures heterogeneity and liquidity preference. Higher belief heterogeneity (increasing p_2 or decreasing q_2) increases Z, as $\partial Z/\partial p_2 > 0$ and $\partial Z/\partial q_2 < 0$, while greater preference for liquidity (decreasing L) decreases Z, as $\partial Z/\partial L > 0$.

Under round financing, the entrepreneur's residual value is

$$\Phi_R = \sum_i p_i(1 - F_i)(1 - \pi)U_i = (1 - \pi)\langle U_i\rangle + Z(1 - \pi)F_1U_1 - (p_2/q_2)(I + q_2J) \quad (16)$$
$$\text{subject to:} \quad I + q_2J = Lq_1F_1(1 - \pi)U_1 + q_2F_2(1 - \pi)U_2,$$
$$(1 - F_1)J = (F_2 - F_1)(1 - \pi)U_2.$$

Thus, determination of the optimal milestone and round financing contracts are essentially the same, except for the extra constraint with round financing. The effects of heterogeneity and liquidity in both (15) and (16) are captured in the Z variable. With homogeneity and no liquidity effect ($p = q$, $L = 1$), Z is zero, and neither milestone nor round financing offers an advantage over the other. With heterogeneity only ($p > q$, $L = 1$), Z is positive; with liquidity preference only ($p = q$, $L < 1$), Z is negative; with both heterogeneity and liquidity ($p > q$, $L < 1$), Z can be positive, negative, or even zero, depending on the degree of disagreement about probabilities and magnitude of preference for liquidity. Except for the $Z = 0$ case, milestone financing offers a strictly better contract than round financing, as the extra degree of flexibility is valuable. We summarize this as follows.

Proposition 3. *Suppose only managerial skill is possible (neither entrepreneurial nor venture capitalist effort), but heterogeneous beliefs about the probability of technological success or venture capitalist liquidity preference are allowed. Then milestone financing is generally preferred to round financing.*

6A. Heterogeneous beliefs, liquidity, managerial skill and venture capitalist effort

This subsection and the next discuss the additional effects of heterogeneous beliefs and venture capitalist liquidity preference that arise when venture capitalist and entrepreneurial efforts are respectively allowed (interaction effects). This subsection allows venture capitalist effort, while the following allows entrepreneurial effort.

Optimal venture capitalist effort under milestone financing implies $b_1 = 1 - \pi + \pi L f_1 U_1/K$, $b_2 = 1 - \pi + \pi f_2 U_2/K$, $c_1 = \pi f_1^2 U_1^2/2K$, and $c_2 = \pi L^2 f_2^2 U_2^2/2K$. Similarly, under round financing, we have $b_1 = 1 - \pi + \pi L F_1 U_1/K$, $b_2 = 1 - \pi + \pi F_2 U_2/K$, $c_1 = \pi F_1^2 U_1^2/2K$, and $c_2 = \pi L^2 F_2^2 U_2^2/2K$. Under milestone financing, the feasibility constraint is

$$I + q_2 J = L q_1 f_1[(1 - \pi)U_1 + \pi f_1 U_1^2 L/2K] + q_2 f_2[(1 - \pi)U_2 + \pi f_2 U_2^2/2K],$$

and the entrepreneur's residual value optimizes

$$\begin{aligned} &\text{Max} \quad \Phi_M = (1 - \pi)\langle U_i\rangle + (\pi/2K)\langle[1 - (1- f_i)^2]U_i^2\rangle - p_1(1 - L)(\pi U_1^2/2K)[1 - (1-f_1)^2] \\ &0 \le f_1 \le f_2 \le 1 \qquad\qquad + Z[(1 - \pi)f_1 U_1 + \pi f_1^2 U_1^2 L/2K] - (p_2/q_2)(I + q_2 J), \qquad (17) \\ &\text{subject to:} \quad I + q_2 J = L q_1 f_1[(1 - \pi)U_1 + \pi f_1 U_1^2 L/2K] + q_2 f_2[(1 - \pi)U_2 + \pi f_2 U_2^2/2K]. \end{aligned}$$

Under round financing, the entrepreneur's residual value is characterized by

$$\Phi_R = (1 - \pi)\langle U_i\rangle + (\pi/2K)\langle[1 - (1 - F_i)^2]U_i^2\rangle - p_1(1- L)(\pi U_1^2/2K)[1 - (1 - F_1)^2]$$

$$+ Z[(1 - \pi)F_1U_1 + \pi F_1^2U_1^2L/2K] - (p_2/q_2)(I + q_2J), \quad (18)$$

subject to: $I + q_2J = Lq_1F_1[(1 - \pi)U_1 + \pi F_1U_1^2L/2K] + q_2F_2[(1 - \pi)U_2 + \pi F_2U_2^2/2K],$

$$J = (F_2 - F_{2D})(1 - \pi)U_2 + (F_2^2 - F_{2D}^2)\,\pi U_2^2/2K,$$

where F_{2D} satisfies the quadratic $F_{2D} = F_1 - F_1\,J/[(1 - \pi)U_2 + \pi F_{2D}U_2^2/K]$. Milestone financing again remains preferred because round financing carries an additional constraint, although the magnitude of the preference may be increased or decreased by heterogeneity or liquidity.

Proposition 4. *Suppose only managerial skill and venture capitalist effort are possible, and heterogeneous beliefs and venture capitalist liquidity preference are allowed. Then milestone financing is preferred to round financing.*

6B. Heterogeneous beliefs, liquidity, managerial skill and entrepreneurial effort

This subsection allows entrepreneurial effort, examining its interaction with heterogeneity and liquidity. No venture capital effort implies $b_i = 1 - \pi$ and $c_i = 0$. Optimal entrepreneurial effort implies $a_i = (1 - f_i)V_i/A$ under milestone financing, or $a_i = (1 - F_1)V_i/A$ under round financing.

Under milestone financing, the entrepreneur's residual value solves

$$\text{Max} \quad \Phi_M = (1 - \pi)\langle U_i\rangle + \langle(1 - f_i^2)V_i^2\rangle/2A + Z[(1 - \pi)f_1U_1 + f_1(1 - f_1)V_1^2/A] - (p_2/q_2)(I + q_2J)$$

$$0 \le f_1 \le f_2 \le 1 \quad (19)$$

subject to: $I + q_2J = Lq_1f_1[(1 - \pi)U_1 + (1 - f_1)V_1^2/A] + q_2f_2[(1 - \pi)U_2 + (1 - f_2)V_2^2/A]$

Under round financing, similar to the earlier argument for (13), F_1 and Φ_R satisfy

$$\Phi_R = (1 - \pi)\langle U_i\rangle + \langle(1 - F_i^2)V_i^2\rangle/2A + Z[(1 - \pi)F_1U_1 + F_1(1 - F_1)V_1^2/A] - (p_2/q_2)(I + q_2J)$$

subject to: $I + q_2F_1J = Lq_1F_1[(1 - \pi)U_1 + (1 - F_1)V_1^2/A] + q_2F_2[(1 - \pi)U_2 + (1 - F_2)V_2^2/A],$

$$F_1 = F_2. \quad (20)$$

As before, milestone financing offers the advantage of flexibility, while round financing has the advantage of less upfront financing. The third terms of Φ_M and Φ_R in (19) and (20) capture the effects of heterogeneity and liquidity preference through Z, and their interaction with flexibility and lower upfront financing through the magnitude of the term. A higher venture capital claim in the unsuccessful state, f_1 or F_1, is more (less) valuable if Z is positive (negative), which occurs when the effect of heterogeneity (liquidity) dominates.

The advantage associated with round financing is that the smaller upfront financing commitment, reflected by I and $(I + q_2F_1J)$ in the constraints of (19) and (20), helps keep F_1 low relative to f_1, improving entrepreneurial incentive. Liquidity preference (decreasing Z) reinforces this incentive effect, while heterogeneity (increasing Z) counteracts it.

The advantage of milestone financing is flexibility. Liquidity preference and heterogeneity may each either reinforce or counteract this flexibility effect, depending upon parameters. To see this, consider the two cases of $p_1V_1^2 > p_2V_2^2$ and $p_1V_1^2 < p_2V_2^2$. Comparing the second terms of Φ_M and Φ_R in (19) and (20), the flexibility of milestone financing allows tilting the contract toward smaller f_1 (relative to F_1) in the first case, and larger f_1 in the second case. In the first case, liquidity preference reinforces the advantage of milestone financing, while heterogeneity counteracts. In the second case, the roles of liquidity preference and heterogeneity are reversed. These results are summarized as follows.

Proposition 5*. The interaction between heterogeneity and/or liquidity preference with entrepreneurial effort leads to the following observations, relative to Proposition 2:*

- *Liquidity preference increases the advantage associated with round financing (i.e., increases entrepreneurial incentive).*
- *Heterogeneity decreases the advantage associated with round financing (i.e., decreases entrepreneurial incentive).*

- *Whether heterogeneity and/or liquidity preference increase or decrease the advantage associated with milestone financing (i.e., contract flexibility) depends upon the parameters.*

7. Conclusion

The two commonly used methods of implementing staged financing for start-up firms are milestone financing, in which the firm receives a commitment for additional injections of financing after certain criteria (milestones) have been reached, and round financing, in which the firm has no explicit commitment, but goes to the venture capital market for additional financing (where it presumably can receive financing if it shows sufficient progress.) This paper has examined the difference in the two contracting methods, concentrating on the effect the contracts can have on the incentives of the contracting parties. We assume both of the contracting parties, entrepreneur and venture capitalist, are able to undertake personally costly effort, which has positive effects on the cash flow of the firm. Although both of these can affect corporate cash flow, the way in which they can do so differs. The entrepreneur, intimately involved with the day-to-day operation of the firm, is assumed to be able to directly affect cash flows with hard work in the early stages of the firm. The venture capitalist has more of a strategic role. In times of crisis, she is able to step in to guide management, help replace management, or offer strategic thinking. This too, can affect cash flow, but through a more indirect method.

In designing an optimal contract, the primary effects we examine are the effects of the contract on the entrepreneur and venture capitalist's incentives. We also examine the effect of differential expectations about the likelihood of success on the real technology and on the venture capitalist's preference for liquid investments on the optimal contract. We work in a context of fully symmetric information and risk-neutral parties. This is a rich context; illustrating, for example, the outcome known as the "living dead," wherein the firm is kept as a going concern, but without involvement from the venture capitalist.

We are able to characterize various sets of parameters for which milestone financing is preferred to round financing, and other sets of parameters for which round financing is preferred to milestone financing. For example, when the role of the venture capitalist is much more important than that of the entrepreneur (in affecting firm fortunes), milestone financing is more effective. When the role of the entrepreneur is much more important, and the real technology is a long-shot, milestone financing is more effective. When the role of the entrepreneur is much more important than that of the venture capitalist and the technological success results in a simple scaling of cash flows and sensitivities to effort, round financing is more effective. When no one's effort is important, the presence of either belief heterogeneity or venture capitalist liquidity preference implies milestone financing is preferable. Thus, the nature of the firm, its cash flows, and sensitivity of its cash flows to entrepreneurial effort, venture capitalist effort, and success of the underlying real technology are important determinants of which of milestone financing and round financing is preferred.

Appendix

Section 3. To determine the optimal contract under milestone financing, note that (3) can be solved with the method of Lagrange multipliers. For the nonbinding $f_1 < f_2$ case, this gives an equation for f_2 in terms of f_1 that can be substituted back into the maximand of (3) to optimize over f_1. For the binding $f_1 = f_2$ case, constraint (2) immediately determines the value of $f_1 = f_2$.

Section 4. To determine entrepreneurial effort a_2 in (6), first employ equation (5). Note that F_{2D} (as a function of a_2) is determined by $F_{2D} = F_1 - F_1 J/[a_2 V_2 + (1 - \pi)U_2 + \pi F_{2D}U_2^2/K]$. Differentiate this with respect to a_2 to determine dF_{2D}/da_2. Next, differentiate (5) with respect to a_2, substituting for dF_{2D}/da_2 from above, to determine dF_2/da_2. Finally, use the first-order condition of (6), substituting for dF_2/da_2 from above, to determine a_2 (as a function of F_1 and F_2).

Section 5B. Combining the two round financing constraints from (12),

$$I + p_2F_1J = F_1(1 - F_1)\langle V_i^2\rangle/A + (1 - \pi)F_1\langle U_i\rangle.$$

We can rewrite (12) as

$$\Phi_R = (1 - F_1^2)\langle V_i^2\rangle/2A + (1 - \pi)\langle U_i\rangle - (I + p_2J), \qquad \text{(A1)}$$

$$\text{subject to:} \quad I + p_2F_1J = F_1(1 - F_1)\langle V_i^2\rangle/A + (1 - \pi)F_1\langle U_i\rangle,$$

$$(1 - F_1)J = (F_2 - F_1)[(1 - F_1)V_2^2/A + (1 - \pi)U_2].$$

Since in (A1), F_2 appears only in the second constraint, F_1 and Φ_R are determined by the first constraint. It follows that the values of F_1 and Φ_R in optimization (13) are the same, although F_2 is not.

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